A comparison between theory and experiment is reported in Fig. 3a for the cylinder. Considering the approximate nature of the theoretical model, the agreement is good for both cases of subcooling and no subcooling. The theory underpredicts the experimental findings somewhat, rather consistently. Two reasons for this fact are speculated. First, the point-contact assumption between the glass beads and the cylinder surface was likely violated somewhat in the experiments, thus increasing the thermal conductivity and yielding higher average heat transfer. Second, probable collapses in the vapor film (partial film boiling) in the back of the cylinder contributed to increasing the average heat flux in the experiments.

Also shown in Fig. 3a is a theoretical prediction for the average heat flux for pool film boiling without subcooling in classical fluids.[7] This average heat flux is lower than what was found in the present study for a cylinder embedded in a bed of glass beads. This result indicates that porous media exhibit a clear potential as heat-transfer augmentation devices if they are made out of a rather conductive material (the glass beads are considerably more conductive than the vapor) and are porous enough to allow for fluid motion inside the solid matrix (the porosity of a bed of spheres, approximately 40%, is reasonable).

Figure 3b shows a comparison between the present theoretical modeling for the sphere and experimental results for the case of zero subcooling reproduced from Ref. 4. No results for subcooled film boiling were available in Ref. 4. An approximate theoretical solution for the average heat flux in classical fluids[7] is also reported. The main conclusion to be drawn from Fig. 3b for the spherical geometry is similar to that of Fig. 3b for the cylinder. For brevity, the discussion is not repeated.

Overall, the experiments exhibited good agreement with the present theoretical predictions for both geometries of interest. Comparison with results for classical fluids indicated that a potential exists in the use of conductive porous materials in heat-augmentation devices.

Acknowledgments

This work was supported by the National Science Foundation and Amoco Research. Thanks are due to Ron Stellman for his help in the numerical calculations.

References

[1]Cheng, P., "Film Condensation Along an Inclined Surface in Porous Medium," *International Journal of Heat and Mass Transfer,* Vol. 24, No. 6, 1981, pp. 983–990.

[2]Cheng, P. and Verma, A. K., "The Effect of Subcooled Liquid on Film Boiling Above a Vertical Heated Surface in a Porous Medium," *International Journal of Heat and Mass Transfer,* Vol. 24, No. 7, 1981, pp. 1151–1160.

[3]Orozco, J., Poulikakos, D., and Gutjahr, M., "Flow Film Boiling from a Sphere and from a Horizontal Cylinder Embedded in a Porous Medium," *Journal of Thermophysics and Heat Transfer,* Vol. 2, No. 4, 1988, pp. 359–364.

[4]Tsung, V. X., Dhir, V. K., and Singh, S., "Experimental Study of Boiling Heat Transfer from a Sphere Embedded in Liquid Saturated Porous Media," *Heat Transfer in Porous Media and Particulate Flows,* HTD-Vol. 46, edited by L. S. Hao, 1985, American Society of Mechanical Engineering, New York, pp. 127–134.

[5]Cheng, P., Chiu, D. K., and Kowk, L. P., "Film Boiling About Two-Dimensional and Axisymmetric Isothermal Bodies of Arbitrary Shape in a Porous Medium," *International Journal of Heat and Mass Transfer,* Vol. 25, No. 8, 1982, pp. 1247–1249.

[6]Orozco, J., Stellman, R., and Gutjahr, M., "Film Boiling Heat Transfer from a Sphere and a Horizontal Cylinder Embedded in a Liquid Saturated Porous Medium," *Journal of Heat Transfer,* Vol. 110, No. 4(A), 1988, pp. 961–967.

[7]Stellman, R., "Study of Film Boiling Heat Transfer from a Cylinder and a Sphere Including Porous Media," M.S. Thesis, Univ. of Illinois at Chicago, IL, 1987.

[8]Gutjahr, M., "Analysis of Natural Convection Film Boiling," M.S. Project, Univ. of Illinois at Chicago, Chicago, IL, 1987.

Emittance of a Finite Spherical Scattering Medium with Fresnel Boundary

Chih-Yang Wu* and Chu-Jeng Wang†
National Cheng Kung University
Tainan, Taiwan, Republic of China

Introduction

THE analysis of radiative transfer within a medium with a reflecting boundary has been a subject of great interest.[1] Recently, several studies were performed to examine the influence of specular and diffuse reflection on radiative transfer with various geometries. Pomraning and Siewert[2] reported the integral form of the equation of radiative transfer for a sphere with a specularly and diffusely reflecting boundary; Thynell and Özisik[3] developed exact integral expressions for radiative transfer in a cylindrical medium with a diffusely and specularly reflecting boundary. Pomraning[4] estimated the emittance from a half-space with a specularly and diffusely reflecting boundary, whereas Lin and Huang[5] studied the directional emittance for an infinite cylinder by the Galerkin method.

Since the radiative heat transfer in semitransparent spherical bodies occurs in many industrial processes such as droplet vaporization and combustion, glass making, and growing of artificial crystals, radiative transfer in a sphere with a Fresnel boundary is of practical importance. This study is intended to show the effects of scattering albedo, refractive index, and optical radius as well as the effects of geometry on the directional emittance for a spherical medium. The ray-tracing technique[2] is adopted to develop the exact integral expressions for radiative transfer in an isotropically scattering sphere with a Fresnel boundary, in which the reflectivity is directionally dependent. The integral form of the equation of radiative transfer is solved by approximating the integral term by a Gaussian quadrature. Exact expressions are then used for the prediction of the directional emittance.

Analysis

The system considered is an absorbing, emitting, and isotropically scattering sphere of a finite optical radius R, which is much greater than radiation wavelength. Assume that the medium is isotropic, homogeneous, isothermal, and in local thermodynamic equilibrium, and that Fresnel reflection is included at the boundary. The equation of radiative transfer and the boundary condition are

$$\mu\frac{\partial I(r,\mu)}{\partial r}+\frac{1}{r}(1-\mu^2)\frac{\partial I(r,\mu)}{\partial\mu}$$
$$+I(r,\mu)=(1-\omega)n^2 I_b(T)$$
$$+\frac{\omega}{2}\int_{-1}^{1} I(r,\mu')\,\mathrm{d}\mu'$$
$$\text{in}\quad 0<r<R,\qquad -1\le\mu\le 1 \tag{1}$$

Received Nov. 21, 1988; revision received March 13, 1989.

*Associate Professor, Department of Mechanical Engineering.
†Graduate Student, Department of Mechanical Engineering.

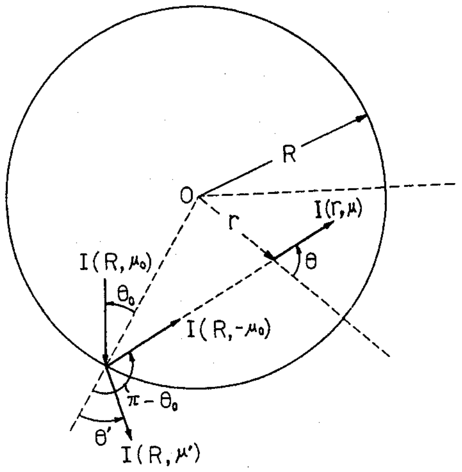

Fig. 1 Geometry and coordinate system.

and

$$I(R,-\mu)=\rho(\mu)I(R,\mu) \tag{2}$$

where I is the radiation intensity, r is the optical variable, $\mu=\cos\theta$ is the cosine of the angle between the radial coordinate and the direction of propagating radiation intensity (see Fig. 1), ω is the scattering albedo, n is the refractive index of the medium relative to that of the surroundings, I_b is the black-body intensity at temperature T, and ρ is the reflectivity of the interface determined by Fresnel relations (see Ref. 1, Chap. 4). Here, the subscript ν, which denotes the spectrally dependent properties, is omitted for simplicity.

We now define the dimensionless total radiation intensity as

$$G(r)=\int_{-1}^{1}\frac{I(r,\mu)\,\mathrm{d}\mu}{n^2 I_b(T)} \tag{3}$$

Solving Eq. (1) formally for the radiation intensity and substituting the resulting expressions into Eq. (3), we can obtain the integral equation for $G(r)$, that is,

$$rG(r)=\int_0^R\left[\frac{\omega}{2}G(x)+(1-\omega)\right]$$

$$\times[E_1(|r-x|)-E_1(r+x)+F_1(r,x)]x\,\mathrm{d}x \tag{4}$$

where

$$F_1(r,x)=4x\int_0^1 U[\mu_0(x,\mu)]\pi^{-1}(r,x,\mu)$$

$$\times\cosh(x\mu)\cosh[\pi(r,x,\mu)]$$

$$\times\exp[-2R\mu_0(x,\mu)]\,\mathrm{d}\mu\quad \text{in}\quad x\leq r \tag{5}$$

$$F_1(r,x)=4r\int_0^1 U[\mu_0(r,\mu)]\pi^{-1}(x,r,\mu)$$

$$\times\cosh(r\mu)\cosh[\pi(x,r,\mu)\exp[-2R\mu_0(r,\mu)]\,\mathrm{d}\mu\quad \text{in} x\geq r \tag{6}$$

with

$$\mu_0(r,\mu)=\left[1-\left(\frac{r}{R}\right)^2(1-\mu^2)\right]^{1/2} \tag{7}$$

$$\pi(x,r,\mu)=[x^2-r^2(1-\mu^2)]^{1/2} \tag{8}$$

$$U(\mu_0)=\frac{\rho(\mu_0)}{1-\rho(\mu_0)\exp(-2R\mu_0)} \tag{9}$$

If ρ is a constant, Eqs. (4-9) reduce to Pomraning and Siewert's[2] formulation for only specular reflection. Applying the ray-tracing technique again, we can obtain the expression for the directional emittance of the spherical medium in terms of $G(r)$ as

$$\epsilon(\theta')=\frac{1-\rho(\mu_0)}{1-(\rho(\mu_0)\exp[-2R\mu_0]}$$

$$\times\int_{R\sqrt{1-\mu_0^2}}^{R}2\left[\frac{\omega}{2}G(x)+(1-\omega)\right]\pi^{-1}(x,R,\mu_0)$$

$$\times\exp(-R\mu_0)\cosh[\pi(x,R,\mu_0)]x\,\mathrm{d}x \tag{10}$$

where

$$\theta'=\sin^{-1}(n\sin\theta_0) \tag{11}$$

Equation (4) is a Fredholm integral equation of the second kind. Crosbie and Pattabongse[6] have shown that a singular Fredholm integral equation can be solved by subtracting the singularity and then approximating the integral term by a Gaussian quadrature. Therefore, Eq. (4) is solved by this technique. Substitution of the results for $G(r)$ into Eq. (10) yields the directional emittance of the sphere.

Results and Discussion

The directional emittance of a sphere has been estimated for some fixed values of the albedo, the refractive index, and the optical radius. Comparisons of the results for various orders of quadrature show that the values of the directional emittance approach constant values once the order of quadratures is large enough. Then, the solutions are assumed to be convergent. Since Eq. (4) is more complicated than the integral equation solved in Ref. 6, we need quadratures of higher orders to generate accurate solutions. Besides, more quadrature points are required for larger values of ω. For example, a quadrature order of 24 is enough for a sphere with $\omega=0.1$, and a quadrature order of 120 is required for a sphere with $\omega=0.9$. This is because the known emission part of the total radiation intensity decreases and the unknown scattering part increases with the increase in albedo.

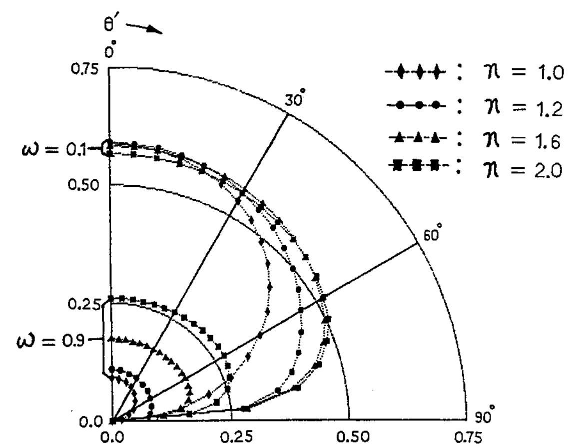

Fig. 2 Variation of directional emittance with the refractive index at $R=0.5$

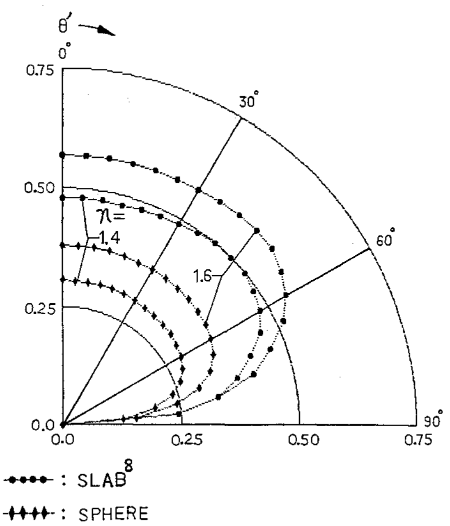

Fig. 3 Comparisons of directional emittance for different geometries at $\omega = 0.95$ (optical thickness = 5.0 for the slab and optical diameter = 5.0 for the sphere).

Figure 2 shows that the refractive index at which the maximum emittance occurs increases with the increase in albedo. This complicated behavior is because of the combined influence of scattering and boundary reflection. This is consistent with the results for planar[8] and cylindrical[5] geometries. Besides, the directional emittance decreases with the increase in the albedo and the optical radius. Such behaviors are also consistent with those for planar and cylindrical geometries.

Since the planar medium is infinite in all directions parallel to the boundaries, the cylindrical medium is infinite only in the axial direction, and the spherical medium is finite in all directions, we can expect that the directional emittance of a sphere is smaller than that of a slab[8] or that of a cylinder[5] with the same physical parameters. The normal emittance of a sphere with $n = 1.6$ is about two-thirds as large as that of a slab with the same refractive index, and the difference between the normal emittance of a slab and that of a sphere increases with the decrease in refractive index, as shown in Fig. 3. This is because the influence of geometry decreases with the increase in surface reflectivity.

References

[1]Siegel, R. and Howell, J. R., *Thermal Radiation Heat Transfer*, 2nd ed., McGraw-Hill, New York, 1981.

[2]Pomraning, G. C. and Siewert, C. E., "On the Integral Form of the Equation of Transfer for a Homogeneous Sphere," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 28, No. 6, 1982, pp. 503–506.

[3]Thynell, S. T. and Özisik, M. N., "Integral Form of the Equation of Transfer for an Isotropically Scattering, Inhomogeneous Solid Cylinder," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 38, No. 6, 1986, pp. 497–503.

[4]Pomraning, G. C., "A Generalized Emissivity Problem," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 32, No. 3, 1984, pp. 191–204.

[5]Lin, J. D., and Huang, J. M., "Radiative Transfer Within a Cylindrical Geometry with Fresnel Reflecting Boundary," *Journal of Thermophysics and Heat Transfer*, Vol. 2, No. 2, 1988, pp. 118–122.

[6]Crosbie, A. L. and Pattabongse, M., "Application of the Singularity-Subtraction Technique to Isotropic Scattering in a Planar Layer," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 34, No. 6, 1985, pp. 473–485.

[7]Crosbie, A. L., "Emittance of a Semi-Infinite Scattering Medium with Refractive Index Greater than Unity," *AIAA Journal*, Vol. 17, No. 1, 1979, pp. 117–120.

[8]Turner, W. D. and Love, T. J., "Directional Emittance of a One-Dimensional Absorbing-Scattering Slab with Reflecting Boundaries," *AIAA Progress in Astronautics and Aeronautics: Thermal Control and Radiation*, Vol. 31, edited by C.-L. Tien, AIAA, New York, 1973, pp. 389–395.

Variable Specific Heat and Thermal Relaxation Parameter in Hyperbolic Heat Conduction

David E. Glass*
Analytical Services and Materials
Hampton, Virginia 23666
and
D. Scott McRae†
North Carolina State University
Raleigh, North Carolina, 27695-7910

Introduction

THE conduction of heat in solids is usually treated mathematically as a diffusion process in which the effect of a thermal disturbance is transmitted throughout the solid with an infinite velocity. However, in some situations, primarily those involving extremely short times or temperatures near absolute zero, the mode of heat conduction is not diffusive (parabolic) but propagative (hyperbolic).[1-3] In the present paper, both the specific heat and the thermal relaxation parameter of the medium are studied parametrically. The effect of varying the specific heat is investigated by assuming a linear relationship with temperature. In addition to studying the effects of a linear specific heat, the thermal relaxation parameter is modeled as a function of time. The relaxation parameter determines how dominant the hyperbolic nature of the heat conduction is. If the relaxation parameter is zero, the hyperbolic heat conduction (HHC) equation reduces to the parabolic equation, and the energy transport is diffusive. As the relaxation parameter increases, the hyperbolic term becomes more important, and the thermal propagation speed decreases. MacCormack's predictor-corrector scheme was used to solve the HHC problems, and an implicit Crank-Nicolson scheme was used to solve the parabolic problems.

Formulation

The medium was taken to be a slab of dimensionless thickness $\eta = 1$. The density and thermal conductivity were assumed to be constant, and the specific heat was assumed to be a function of the local temperature, and the thermal relaxation parameter was assumed to be a function of time. The HHC equation results from the use of the non-Fourier heat flux equation given in dimensionless form as

$$\tau \frac{\partial Q}{\partial \xi} + 2Q + K \frac{\partial \theta}{\partial \eta} = 0 \qquad (1)$$

where Q is the dimensionless conduction heat flux, θ is the dimensionless temperature, τ is the dimensionless thermal relaxation parameter, and K is the dimensionless thermal conductivity. In addition, the dimensionless distance and time are given, respectively, by η and ξ. Clearly, when the relaxation parameter τ is equal to 0, the non-Fourier heat flux equation reduces to the classical Fourier heat flux equation. Although the heat flux equations are different for the hyperbolic and parabolic formulations, the energy equation remains un-

Received Oct. 31, 1988; presented as Paper 89-0317 at the AIAA 27th Aerospace Sciences Meeting, Reno, NV, Jan. 9–12, 1989.

*Research Engineer. Member AIAA.

†Associate Professor. Department of Mechanical Engineers. Member AIAA.